MATH 1700: SECTION 10.4: THE OTHER CIRCULAR FUNCTIONS

THE CIRCULAR FUNCTIONS:

If θ is graphed in standard position and $P(x, y)$ is the where the terminal side of θ intersects the Unit Circle:

- The **sine** of θ, denoted $\sin(\theta)$, is defined by $\sin(\theta) = y$.
- The **cosine** of θ, denoted $\cos(\theta)$, is defined by $\cos(\theta) = x$.
- The **tangent** of θ, denoted $\tan(\theta)$, is defined by $\tan(\theta) = \dfrac{y}{x}$, provided $x \neq 0$.
- The **secant** of θ, denoted $\sec(\theta)$, is defined by $\sec(\theta) = \dfrac{1}{x}$, provided $x \neq 0$.
- The **cosecant** of θ, denoted $\csc(\theta)$, is defined by $\csc(\theta) = \dfrac{1}{y}$, provided $y \neq 0$.
- The **cotangent** of θ, denoted $\cot(\theta)$, is defined by $\cot(\theta) = \dfrac{x}{y}$, provided $y \neq 0$.

Of the six circular functions, only sine and cosine are defined for all angles θ. Since $x = \cos(\theta)$ and $y = \sin(\theta)$, it is customary to rephrase the remaining four circular functions in terms of sine and cosine:

RECIPROCAL AND QUOTIENT IDENTITIES:

- $\sec(\theta) = \dfrac{1}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$, $\sec(\theta)$ is undefined.
- $\csc(\theta) = \dfrac{1}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$, $\csc(\theta)$ is undefined.
- $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$, $\tan(\theta)$ is undefined.
- $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$, $\cot(\theta)$ is undefined.
- $\cot(\theta) = \dfrac{1}{\tan(\theta)}$, provided $\tan(\theta) \neq 0$; if $\tan(\theta) = 0$, $\cot(\theta)$ is undefined.

CIRCULAR FUNCTION VALUES OF COMMON ANGLES

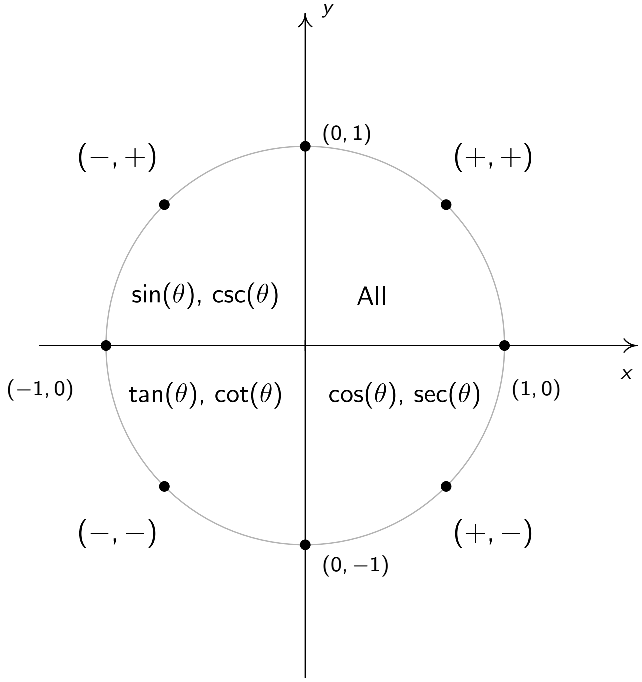
θ(degrees)	θ(radians)	$\sin(\theta)$	$\cos(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\tan(\theta)$	$\cot(\theta)$
0°	0	0	1	1	undefined	0	undefined
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$	$\sqrt{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	undefined	1	undefined	0

GENERALIZED REFERENCE ANGLE THEOREM: The values of the circular functions of an angle, if they exist, are the same, up to a sign, of the corresponding circular functions of its reference angle.

More specifically, if α is the reference angle for θ, then:

$$\sin(\theta) = \pm\sin(\alpha),\ \cos(\theta) = \pm\cos(\alpha),\ \tan(\theta) = \pm\tan(\alpha)$$
and
$$\sec(\theta) = \pm\sec(\alpha),\ \csc(\theta) = \pm\csc(\alpha),\ \cot(\theta) = \pm\cot(\alpha)$$

where the choice of the $(\pm)$ depends on the quadrant in which the terminal side of θ lies.



Postive Circular Functions

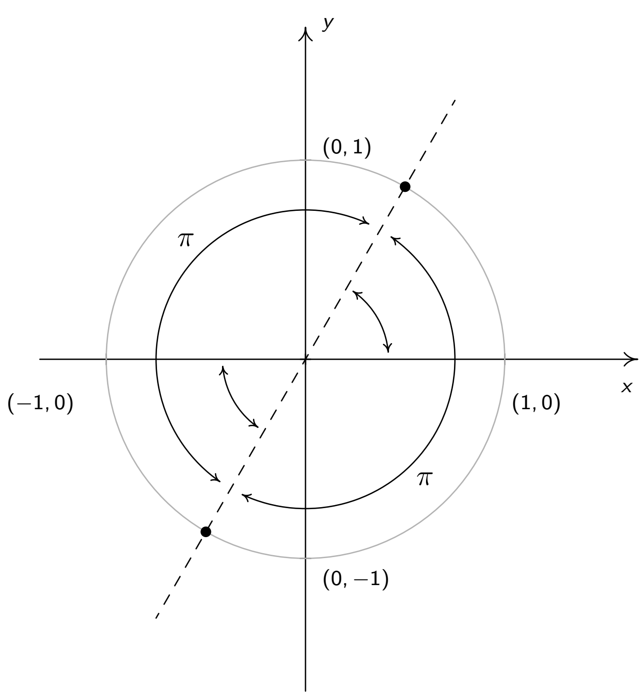
EXAMPLE 1: Find the exact value of the following, if it exists:

1. $\sec(60^\circ)$
2. $\csc\left(\frac{7\pi}{4}\right)$
3. $\tan(225^\circ)$
4. $\cot\left(-\frac{7\pi}{6}\right)$

EXAMPLE 2: Find all angles which satisfy the given equation.

1. $\sec(\theta) = 2$
2. $\csc(\theta) = -\sqrt{2}$
3. $\tan(\theta) = \sqrt{3}$
4. $\cot(\theta) = -1.$

THE PERIOD OF TANGENT AND COTANGENT:



The period of $\tan(\theta)$ and $\cot(\theta)$ is π

BEYOND THE UNIT CIRCLE:

If θ is graphed in standard position and $Q(x, y)$ is the where the terminal side of θ intersects $x^2 + y^2 = r^2$:

- $\sin(\theta) = \dfrac{y}{r} = \dfrac{y}{\sqrt{x^2 + y^2}}$
- $\cos(\theta) = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2 + y^2}}$
- $\tan(\theta) = \dfrac{y}{x}$, provided $x \neq 0$.
- $\sec(\theta) = \dfrac{r}{x} = \dfrac{\sqrt{x^2 + y^2}}{x}$, provided $x \neq 0$.
- $\csc(\theta) = \dfrac{r}{y} = \dfrac{\sqrt{x^2 + y^2}}{y}$, provided $y \neq 0$.
- $\cot(\theta) = \dfrac{x}{y}$, provided $y \neq 0$.

EXAMPLE 3:

1. If the terminal side of θ, when plotted in standard position, contains the point $Q(3, -4)$, find the values of the six circular functions of θ.

2. If θ is a Quadrant IV angle with $\cot(\theta) = -4$, find the values of the five remaining circular functions of θ.

3. Find $\sin(\theta)$, where $\sec(\theta) = -\sqrt{5}$ and θ is a Quadrant II angle.

4. Find $\cos(\theta)$, where $\tan(\theta) = 3$ and $\pi < \theta < \frac{3\pi}{2}$.

HOMEWORK: Section 10.4: Page 877: 1 - 67 odd, 70-81*